CONVECTIVE DIFFUSION ACCOMPANYING EVAPORATION IN A MACROCAPILLARY

V. V. Levdanskii and N. V. Pavlyukevich

UDC 532.72:532.66

The slip velocity is taken into account in the derivation of expressions for the pressure gradient and mass velocity of a vapor–gas mixture undergoing evaporation in a single capillary.

A description of the drying process requires an investigation of evaporation in a single capillary where there is not only mutual diffusion of vapor and air, but also a convective flow of the mixture. We will assume that the ratio of the mean free path l of the molecule to the capillary radius r_0 is not very small relative to unity (slip regime [1]). In this case we can expect that owing to the presence of friction, and also of viscous, thermal, and diffusive slip, the mass transfer process in the case of evaporation in a capillary will differ, generally speaking, from mass transfer in the case of evaporation into a semiinfinite space.

The condition for the slip velocity at the boundary of a body in a flow has the form (we will assume that the accommodation coefficient is unity) [2]

$$v|_{r=r_0}=\frac{\frac{\mu}{2}\cdot\frac{\partial v}{\partial r}\Big|_{r_0}+\frac{1}{5(2\pi T)^{1/2}}\left[\frac{\lambda_{12}^{(1)}}{(R^{(1)})^{1/2}}+\frac{\lambda_{12}^{(2)}}{(R^{(2)})^{1/2}}\right]\frac{dT}{dx}}{\rho\left(\frac{T}{2\pi}\right)^{1/2}\left[c(R^{(1)})^{1/2}+(1-c)(R^{(2)})^{1/2}\right]}+\frac{D_{12}\left[(R^{(1)})^{1/2}-(R^{(2)})^{1/2}\right]\frac{dc}{dx}}{c(R^{(1)})^{1/2}+(1-c)(R^{(2)})^{1/2}},\qquad(1)$$

where $\lambda_{12}^{(1)}$ and $\lambda_{12}^{(2)}$ are coefficients which depend on the collisions of molecules of a given component with molecules of the same species or with molecules of another component [2].

In expression (1) the first term on the right represents the viscous slip, the second represents the thermal slip (we will assume that the temperature gradient along the capillary is known), and the third represents the diffusive slip. We note that in the case of diffusion of components with approximately equal molecular weights the velocity of diffusive slip is low.

The mass velocity of a vapor–gas mixture in a cylindrical capillary will satisfy the Poiseuille equation of motion with boundary condition (1). We assume that in a sufficiently fine and long capillary the pressure, density, and concentration can be regarded as constant over the cross section. We also neglect the change in velocity and viscosity coefficient along the capillary. Then, for the mean velocity over the cross section we have

$$\bar{v}=-\frac{r_0^2}{8\mu}\cdot\frac{dp}{dx}+v_{sl}=-\frac{r_0^2}{8\mu}\cdot\frac{dp}{dx}-K\frac{r_0}{2\mu}\cdot\frac{dp}{dx}+v'_{\text{ск}}=-\left(\frac{r_0^2}{8\mu}+K\frac{r_0}{2\mu}\right)\frac{dp}{dx}+N\frac{dT}{dx}+M\frac{dc}{dx},\qquad(2)$$

where

$$K=\frac{1}{2}\,\frac{\mu}{\rho\left(\frac{T}{2\pi}\right)^{1/2}\left[c(R^{(1)})^{1/2}+(1-c)(R^{(2)})^{1/2}\right]},$$

$$N=\frac{\frac{1}{5(2\pi T)^{1/2}}\left[\frac{\lambda_{12}^{(1)}}{(R^{(1)})^{1/2}}+\frac{\lambda_{12}^{(2)}}{(R^{(2)})^{1/2}}\right]}{\rho\left(\frac{T}{2\pi}\right)^{1/2}\left[c(R^{(1)})^{1/2}+(1-c)(R^{(2)})^{1/2}\right]};$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 20, No. 6, pp. 1093-1095, June, 1971. Original article submitted August 7, 1970.

© 1973 Consultants Bureau, a division of Plenum Publishing Corporation, 227 West 17th Street, New York, N. Y. 10011. All rights reserved. This article cannot be reproduced for any purpose whatsoever without permission of the publisher. A copy of this article is available from the publisher for $15.00.

$$M = \frac{D_{12}\left[(R^{(1)})^{1/2} - (R^{(2)})^{1/2}\right]}{c\,(R^{(1)})^{1/2} + (1-c)\,(R^{(2)})^{1/2}}.$$

In formula (2) the term corresponding to viscous slip is expressed in terms of the total pressure gradient [1].

For evaporation in a capillary we have to use another condition, viz., that the total (diffusive and convective) flow of air through the cross section of the capillary is zero (we neglect thermal diffusion and pressure diffusion):

$$J_2 = -D_{12}\frac{n^2 m_1 m_2}{\rho}\cdot\frac{d\left(\frac{p_2}{p}\right)}{dx} + \rho_2\bar{v} = 0, \tag{3}$$

whence

$$\frac{d\left(\frac{p_2}{p}\right)}{dx} = \frac{\rho_2\rho}{n^2 m_1 m_2 D_{12}}\,\bar{v}. \tag{4}$$

Using (4), and also the obvious relationship

$$\frac{d\left(\frac{p_2}{p}\right)}{dx} = \frac{1}{p}\cdot\frac{dp_2}{dx} - \frac{p_2}{p^2}\cdot\frac{dp}{dx},$$

we obtain from (2) an expression for the mean velocity of the mixture

$$\bar{v} = \frac{\frac{D_{12}}{p}\cdot\frac{m_1}{m}\cdot\frac{1}{c_2'}\cdot\frac{dp_2}{dx} - \alpha c_2'\left(N\frac{dT}{dx} + M\frac{dc}{dx}\right)}{1-\alpha c_2'}. \tag{5}$$

In a similar way we find an expression equivalent to (5):

$$\bar{v} = \frac{-\frac{D_{12}}{p}\cdot\frac{m_1}{m}\cdot\frac{1}{c_2'}\cdot\frac{dp_1}{dx} + \alpha c_1'\left(N\frac{dT}{dx} + M\frac{dc}{dx}\right)}{1+\alpha c_1'}, \tag{6}$$

where

$$\alpha = \frac{D_{12}}{p}\cdot\frac{m_1}{m}\cdot\frac{1}{c_2'}\left(\frac{r_0^2}{8\mu} + K\frac{r_0}{2\mu}\right)^{-1}. \tag{7}$$

The presence of mass velocity accompanying evaporation in a capillary implies a longitudinal pressure gradient, which we determine from (2) and (6), assuming that $(\alpha c_1')^2 \ll 1$:

$$\frac{dp}{dx} = (1-\alpha c_1')\left(\frac{r_0^2}{8\mu} + K\frac{r_0}{2\mu}\right)^{-1}\left(\frac{D_{12}}{p}\cdot\frac{m_1}{m}\,\frac{1}{c_2'}\cdot\frac{dp_1}{dx} + N\frac{dT}{dx} + M\frac{dc}{dx}\right). \tag{8}$$

Thus, in the case of evaporation in a capillary there is a total pressure gradient (8), and the mean velocity of the mixture (5) [or (6)], generally speaking, differs from the Stefan velocity, which has the form [3]:

$$\bar{v} = -\frac{D_{12}}{p}\cdot\frac{m_1}{m}\cdot\frac{1}{1-c_1'}\cdot\frac{dp_1}{dx}. \tag{9}$$

These differences disappear, however, i.e., p = const, and formula (5) becomes the same as (9):

1) when the capillary radius is large enough, i.e., $r_0 \to \infty$ ($\alpha \to 0$);

2) when the effect of surface friction in the capillary is compensated by the effect of thermal and diffusive slip, i.e.,

$$-\frac{D_{12}}{p}\cdot\frac{m_1}{m}\cdot\frac{1}{c_2'}\cdot\frac{dp_1}{dx} = N\frac{dT}{dx} + M\frac{dc}{dx}.$$

NOTATION

$\bar{v}$	is the mean mass velocity of mixture;
v_{sl}	is the slip velocity;
$\rho = nm$	is the density of mixture;
T	is the temperature of mixture;
p_1, p_2	are the partial pressures of vapor and air, respectively;
c	is the mass concentration of vapor;
$c_1' = p_1/p$, $c_2' = p_2/p$	are the molar concentrations of vapor and air;
m	is the molecular weight of mixture;
$R^{(1)}$, $R^{(2)}$	are the gas constants of vapor and air;
μ	is the coefficient of viscosity of mixture;
λ	is the thermal conductivity of mixture;
λ_1, λ_2	are the thermal conductivities of vapor and air;
x	is the coordinate along capillary;
D_{12}	is the diffusion coefficient.

LITERATURE CITED

1. M. Devienne, Flow and Heat Transfer of Rarefied Gases [Russian translation], IL (1962).
2. V. G. Leitsina and N. V. Pavlyukevich, Inzh.-Fiz. Zh., 12, No. 3 (1967).
3. A. V. Lykov, Theory of Drying [in Russian], Energiya (1968).